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If there can be found a value for n that will render $(n-1)(128n^3-8n-8)(256n^6-256n^4+63n^2-2n-1)+(256n^6-288n^4-32n^3+65n^2+2n+1)^2=\square$ then the five numbers are $(n-1)$, $(n+1)$, $4n$, $4n(4n^2-1)$, and $[(256n^6-256n^4+63n^2-2n-1)(128n^3-8n-8)]/(256n^6-288n^4-32n^3+65n^2+2n+1)^2$; but life is too short to attempt this.

Dr. Zerr says, "I have not been able to find the five numbers."

If you get no answers to problem 71, I can give Legendre's (improved), and it is a fractional answer for the fifth number. Mr. Wilkes has found four numbers, as follows: $n-1$, $n+1$, $4n$, $16n^3-4n$. Taking these for four of them then I can prove that if there is another integral number it must end in 0.

I have investigated, up to numbers having over five thousand digits, without finding any to answer. A. H. BELL.

I have given considerable time to problem 71, but as yet have failed to obtain five numbers. I have found general values for three such numbers, viz., m, $n(mn\pm2)$, and $(n+1)[(n+1)m\pm2]$. I have also found partial general values for four such numbers, viz., m, $m\pm2$, $4(m\pm1)$, and $16(m\pm1)^3-4(m\pm1)$; but one of the sets of four numbers that I obtained by inspection, can not be obtained by this formula. The set is 1, 8, 15, 528. M. A. GRUBER.

MISCELLANEOUS.

63. Proposed by **F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.**

Show that the path of a projectile moving with a constant velocity is an inverted catenary of equal strength.

Solution by **GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.**

The differential equations are, obviously,

$$\frac{d^2y}{dt^2}=-g,\quad \frac{ds}{dt}=c.$$

Whence, $\dfrac{dy}{dt}=-gt+k$, $\dfrac{dx}{dt}=\sqrt{c^2-(gt-k)^2}$, $x+k'=-\dfrac{1}{g}\cos^{-1}\dfrac{gt-k}{c}$.

By division, $\dfrac{dy}{dx}=\dfrac{-\cos g(x+k')}{\sqrt{c^2-c^2\cos^2 g(x+k')}}=-\cot g(x+k')$,

$$\text{or, } \cot^{-1}\frac{dy}{dx}=-g(x+k').$$

Differentiation gives

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{-1}\frac{dy^2}{dx^2}=g.$$

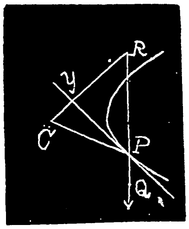
This is the differential equation of the catenary of uniform strength. See Routh's Statics, page 329, or Minchin's Statics, page 315.

64. **Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

Find the caustic by reflection of an hyperbola, the bright point being the center.

Solution by the PROPOSER.

Let RPQ be a reflected ray, Q a point of the caustic where RPQ touches the caustic, R the corresponding point on the secondary caustic. Then RQ is the radius of curvature of the secondary caustic, that is, $RQ=2\rho$, where ρ is the radius of curvature of the locus of y, the foot of the perpendicular from the center C on the tangent at P. Let δ be the perpendicular from C on the tangent at y to the first pedal, then $\delta r=p^2$ (Williamson's Differential Calculus, Art. 188, page 228, Sixth Edition).



Differentiating, $r\frac{d\delta}{dp}+\delta\frac{dr}{dp}=2p$. But $\rho=\frac{pdp}{d\delta}$; and since $p^2(r^2-a^2+b^2)=a^2b^2$ from the hyperbola.

$$\therefore \frac{rdr}{dp}=-\frac{a^2b^2}{p^3}. \quad \therefore \frac{r}{\rho}-\frac{p^2}{r}.\frac{a^2b^2}{p^4r}=2, \text{ or } \frac{a^2b^2}{p^2r^2}=\frac{r}{\rho}-2.$$

Let (m, n) be the coördinates of R, ψ the eccentric angle of P, (x, y) the coördinates of Q. Then

$$\frac{QR}{QP}=\frac{x-m}{x-a\sec\psi}=\frac{y+n}{y-b\tan\psi}=\frac{2\rho}{2\rho-r}.$$

In triangle CPR, $Cy=Ry$, $CP=RP=r$.

$$\therefore \frac{x-m}{x-a\sec\psi}=\frac{y+n}{y-b\tan\psi}=-\frac{2p^2r^2}{a^2b^2}=\frac{2r^2}{a^2-r^2-b^2}.$$

$$\therefore x(a^2-b^2-3r^2)=m(a^2-b^2-r^2)-2r^2a\sec\psi.$$

$$y(a^2-b^2-3r^2)+n(a^2-b^2-r^2)+2r^2b\tan\psi=0.$$

Now $m^2+n^2=4p^2$, $(m-a\sec\psi)^2+(n+b\tan\psi)^2=r^2$.

$$\therefore m=\frac{2ab^2\sec\psi}{a^2\tan^2\psi+b^2\sec^2\psi},\ n=\frac{2a^2b\tan\psi}{a^2\tan^2\psi+b^2\sec^2\psi}.$$

But $m(a^2-b^2-r^2)=-m(r^2-a^2+b^2)=-m(a^2\tan^2\psi+b^2\sec^2\psi)=-2ab^2\sec\psi$.